DE2.3 Electronics 2 for Design Engineers

SOLUTIONS
Tutorial Sheet 6 – Feedback Control
(Lectures 14 - 17)

* indicates level of difficulty

1.* Bring the second summer to the left of G1 and combine it with the first; then move the middle tap point to the right of G2. This produces the equivalent block diagram:

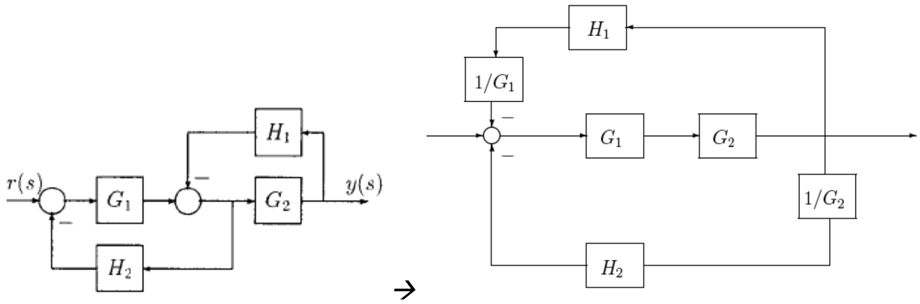
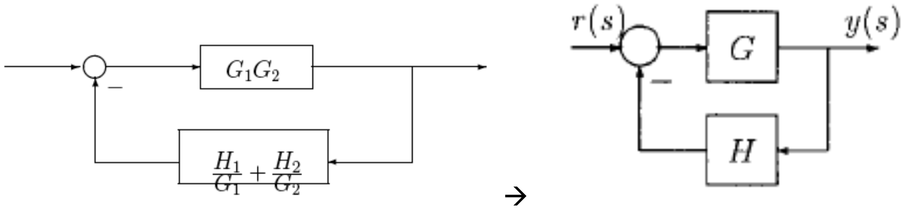


Figure Q1

The feedback paths are in parallel and can be combined:



2.* For the close-loop system, the system transfer function is:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{\frac{100}{\tau s + 1}}{1 + \frac{100}{\tau s + 1}} = \frac{\frac{100}{101}}{1 + \frac{\tau}{101}s}$$

Therefore, the time constant of the close loop system is $\frac{\tau}{101} = 3/101$, i.e. much faster!

3.** (i) Loop transfer function:

$$L(s) = P(s)C(s) = \frac{k_p bs + k_i b}{s(s + a)} = \frac{n_L(s)}{d_L(s)}$$

(ii) The transfer function of the closed loop system is:

$$\frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)} = \frac{n_L(s)}{d_L(s) + n_L(s)} = \frac{b(k_p s + k_i)}{s^2 + (a + bk_p)s + bk_i}$$

(iii) The denominator provides the system's new characteristic polynomial:

$$d_L(s) + n_L(s) = s^2 + (a + bk_p)s + bk_i$$

Equating the coefficients of this equation to that of $s^2 + 2\zeta\omega_0 s + {\omega_0}^2$, we get:

$$k_p = \frac{2\zeta\omega_0 - a}{b}$$

$$k_i = \frac{{\omega_0}^2}{b}$$

Therefore, k_i directly affects the resonant frequency (i.e. the stiffness of the system), and k_p affects the damping factor of the system.

4.*** This question shows how feedback help to make operational amplifier really useful.

At the amplifier input, $\upsilon_+ = 0$ and

$$\upsilon_- - \upsilon_i = \frac{R_1}{R_1 + R_2}(\upsilon_0 - \upsilon_i). \quad \text{(potential divider)}$$

So, at its output, the voltage is

$$\upsilon_0 - \upsilon_n = -A\upsilon_- = -A\left(\upsilon_i + \frac{R_1}{R_1 + R_2}(\upsilon_0 - \upsilon_i)\right).$$

Rearrange the terms

$$\upsilon_0 = \frac{-AR_2}{AR_1 + R_1 + R_2}\upsilon_i + \frac{R_1 + R_2}{AR_1 + R_1 + R_2}\upsilon_n = G_1\upsilon_i + G_2\upsilon_n.$$

Equivalently

$$\upsilon_0 = \frac{A}{1 + \frac{AR_1}{R_1+R_2}} \frac{(-R_2)}{R_1 + R_2}\upsilon_i + \frac{1}{1 + \frac{AR_1}{R_1+R_2}}\upsilon_n = G_1\upsilon_i + G_2\upsilon_n$$

which is represented by the block diagram in (b).

Equivalently

$$\upsilon_0 = \frac{A}{1 + \frac{AR_1}{R_1+R_2}} \frac{(-R_2)}{R_1 + R_2}\upsilon_i + \frac{1}{1 + \frac{AR_1}{R_1+R_2}}\upsilon_n = G_1\upsilon_i + G_2\upsilon_n$$

which is represented by the block diagram in (b).

(i) Given that R1=10kΩ, R2 = 100Ω, A=10^4:

$$G_1 = \frac{A}{1+\frac{AR_1}{R_1+R_2}} \frac{(-R_2)}{R_1+R_2} = \frac{10^4}{1+\frac{10^4 10}{10.1}} \frac{-0.1}{10.1} \approx -0.01$$

$$G_2 = \frac{1}{1+\frac{AR_1}{R_1+R_2}} = \frac{1}{1+\frac{10^4 x10}{10.1}} \approx 10^{-4}$$

Conclusions:

1. Signal gain is determined by R2/R1 ratio as you found out last year from my lectures on inverting amplifiers using op amp. This is due to the negative feedback used in the circuit.
2. The disturbance (noise injected into the circuit, say, from power supply) is reduced by a factor = A, the op amp gain.

(ii)

The sensitivities, for nominal values $R_1 = 10K\Omega, R_2 = 100\Omega, A = 10^4$ with respect to a 10% change, are obtained by differentiating G_1 with respect to, in turn, R_1, R_2 and A:

$$\frac{dG_1}{dR_1} = \frac{AR_2(A+1)}{[(A+1)R_1 + R_2]^2}.$$

So, in terms of differentials

$$\frac{dG_1}{G_1} = -\frac{(A+1)R_1}{AR_1 + R_1 + R_2}\,\frac{dR_1}{R_1} \approx -10\%.$$

Similarly for varying R_2 and A,

$$\frac{dG_1}{G_1} = \frac{AR_1 + R_1}{AR_1 + R_1 + R_2}\,\frac{dR_2}{R_2} \approx 10\%$$

$$\frac{dG_1}{G_1} = \frac{R_1 + R_2}{AR_1 + R_1 + R_2}\frac{dA}{A} \approx 0.001\%.$$

Note the insensitivity to changes in A.

This shows that using feedback, we reduce the sensitivity to of the overall gain G1 to A to near zero. However, any change in resistor value is directly reflected in the system gain.